The 6d (1,0) and (2,0) SCFTs

Kimyeong Lee
KIAS

High1-2014, KIAS-NCTS Joint Workshop
February 2014

Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM [arXiv:1307.7660] The general M5-brane superconformal Index

Hee-Cheol Kim, KM [arXiv:1210.0853] M5 brane theories on R x CP2

Hee-Cheol Kim, Seok Kim, Eunkyung Ko, KM [arXiv:1110.2175] On instantons as KK modes of M5 branes

Stefano Bolognesi, KM [arXiv:1105.5073] 1/4 BPS string junctions and N3 problem in 6-dim conformal field theories

String/M Theory

- Fundamental Formulation
- From 11-dimension to 4-dimension
- AdS-CFT Correspondence

- D3 branes, 4d N=4 SYM, $AdS_5 \times S^5$

- M2 branes, 3d N=8 ABJM, $AdS_4 \times S^7$

- M5 branes, 6d (2,0) Theory, $AdS_7 \times S^4$

6d (2,0) Superconformal Theories

- A, D, E type: type IIB on $R^{1+5} \times C^2/\Gamma_{ADE}$
  - A_{N-1}, D_N type: N M5 branes, N M5 (+OM5)
- superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times USp(4)_R$
- fields: B, $\Phi_{I\ (I=1,2,3,4,5)}$, Ψ
  - selfdual strength $H=dB={}^*H$, purely quantum $\hbar=1$

- We do not know how to write down the theory for nonabelian case.
  - Sorokin, Chu, Lambert, Papageorgakis, ...
  - covariant derivative?
- N^3 degrees of freedom

- Calculate something exact on (2,0) theories?

6d (1,0) Superconformal Field Theories

- Many types of Theories:
- (1) 6d E8 (1,0) Superconformal Field Theory ;
  - M5 branes near one of E8 Walls of M theory on $R^{1+9} \times S^1/Z_2$
  - Tensor multiplet+ hypermultiplet......
- (2) Mixed with tensor multiplets, gauge multiplets, matter hypermultiplets
  - hypermultiplets coupled to vectors
  - hypermultiplets coupled to tensors
- Nonabelian tensor theories, N^3 degrees of freedom
- Witten, Ganor, Hanany, Seiberg, Duff, Lu, Pop, Morrison, Aspinwall, Berkooz, Leigh, Schwarz, brunner Karch, Zaffarony, How, Sezgin, West, Pasti, Sorokin, Tonin, Bandos,
-

5-dim Approach to 6d (2,0) theories

- Instantons= KK modes
  - threshold bound states of k instantons for kk momentum k.
- Douglas, Lambert, Papageorgakis
- 5d N=2 super Yang-Mills theories could be complete when all non perturbative effects are included

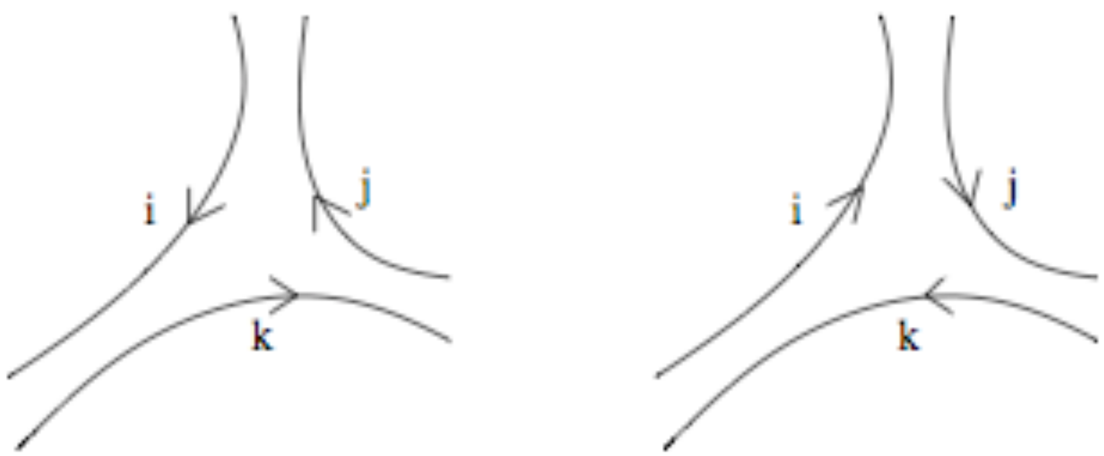
  Z. Bern, J. J. Carrasco, L. J. Dixon, M. R. Douglas, H. Johansson, M. von
  - Perturbative approach: problem in higher order (6-loop)
  - incomplete? (Need higher order operators to complete the theory?)
  - strong coupling limit = 6d theory
- BPS object counting
  - dyonic instanton counting = S-dual of elliptic genus of selfdual strings in Coulomb phase of (2,0) theories (Nekrasov-partition function..)

    Berkooz, Rozali, Seiberg 1997,Berkooz, Douglas1996
  - DLCQ of the (2,0) theory = (gauge invariant operators..)

    Hee-Cheol Kim, Seok Ki, E. Koh, KL, Sungjay Lee:[arXiv:1110,2175]
- monopole string junctions
  - selfdual string junctions in the Coulomb phase of 6d (2,0) theories

More Lessons from 5d SYM

- 1/4 BPS selfdual string junction in the Coulomb phase of 6d (2,0) theory
  - possible solution for N^3 degrees of freedom.
  - The rough entropy calculation in the Coulomb phase seems to work.



KL, Ho-UngYee:0606150
Stefano Bolognesi,KL:1105.5073

BPS Junction Math for A(DE)

- dimension of A_{N-1}: $d=N^2-1$
- rank of A_{N-1}: $r=N$
- Coxeter number= number of roots/rank: $h=N$
  - Coxeter=Dual Coxeter for simple laced groups
- Anomaly coefficient : $c = dh/3 = N(N^2-1)/3$
- Relation:

$$c= N(N^2-1)/3= N^2-N + N(N-1)(N-2)/3$$

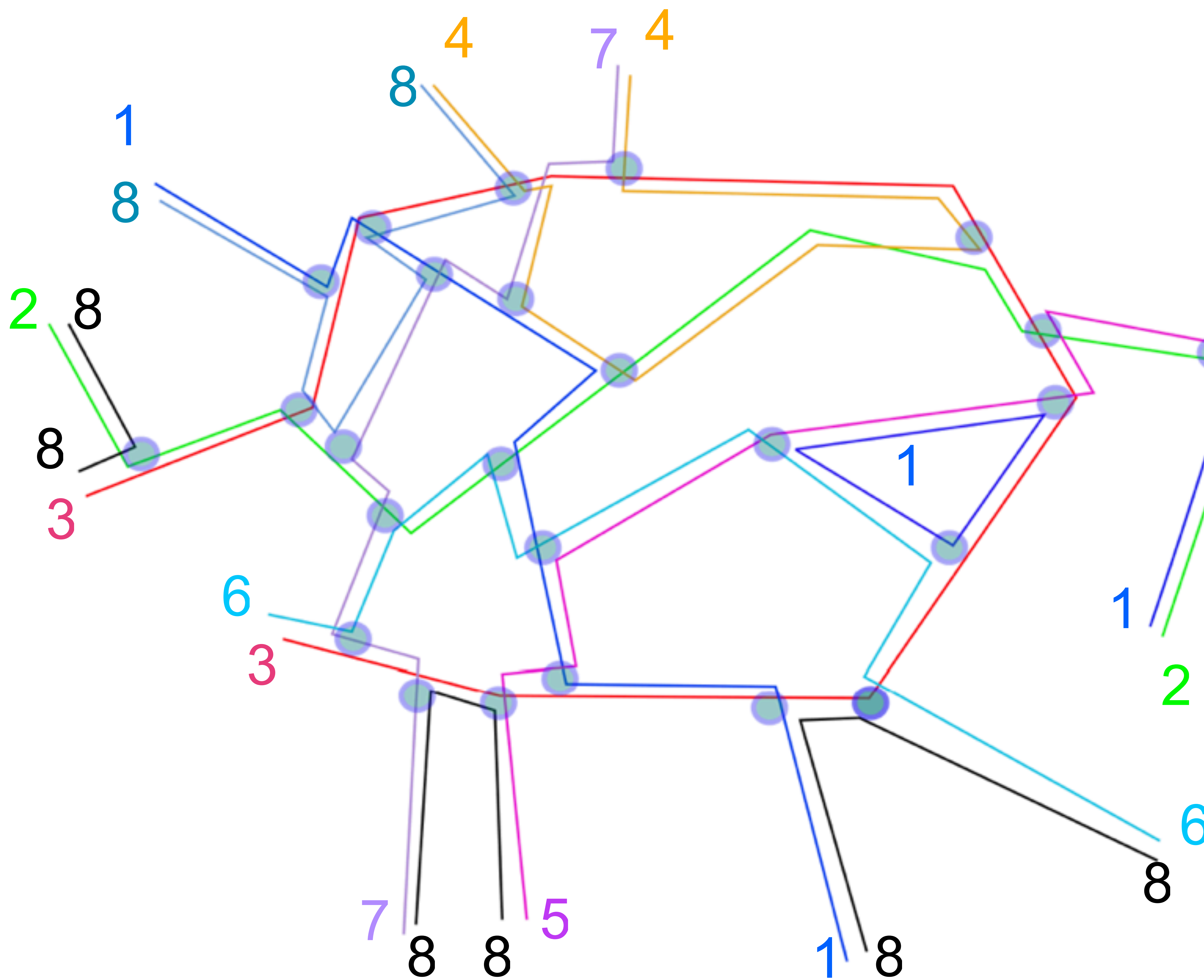
- # of roots= ij selfdual strings=# of roots: $N(N-1)$
- # of SU(3) imbedding= ijk of BPS (anti)junctions: $N(N-1)(N-2)/3$
- True for ADE algebras

High Temperature in Coulomb Phase

- Micro-canonical
- Massless on N M5 branes: O(N)
- Loops of self-dual strings excitations: $O(N^2)$
- Beyond the Hagedorn temperature
- Webs of junctions and anti-junctions: $O(N^3)$
- Excitations of webs of tensionless strings with junctions act as atoms.

  - N^3 degrees of freedom

Nonzero Temperature in Symmetric Phase



5-dim Approach to 6d (1,0) Theories

- 6d E_8 SCFT compactified on $R^{1+4} \times S^1$
  - 5-dim non-Lagrangian theory + KK modes
- With Wilson-loop along the flavor direction which breaks E_8 to SO(16)
- 5d N=1 susy USp(N) gauge theory with N_f=8 fundamental hyper + antisymmetric hyper
  - strongly coupled limit = 6d theory ('KK modes' should be captured by instanton)
- 5d N=1 susy USp(N) theory with Nf $\leqq$ 7 fundamental hyper+ antisymmetric hyper
  - strongly coupled limit = 5d supersymmetric theory with E_{Nf+1} global symmetry
  - the index calculation on $S^1 \times S^4$ has shown this recently by us and others.
- It is not known whether the instantons of the 5d version of 6d SCFT (H^2+ $d\Phi^2$ + ΦF^2+BFF+..) captures KK modes 6d theory
-

6d (2,0) Theories

- Difficulties with nonabelianization of the B field and its strength H=dB
  - after the torus compactification to 4d, instantons are magnetically charged KK modes.
  - its electric dual is the adjoint fields of the 5d theory kk modes from 5d to 4d
  - A explicit construction of a local field theory including the nonabelian 2-form field would leads a 4d local field theory with nonabelian electric and magnetically charged fields. (Besides abelian case, it is not known yet.)
- Generalize ABJM to M5 brane theory
  - Mode out by R^8/Z_k
  - Weak coupling limit
  - No fixed point
- AdS_7 x S^4/Z_k (Tomasiello): 6d theory with D6 and $\underline{D6}$: still 6d theory with H=*H

6d (2,0) Theory on R x S^5

- Radial quantization
- S^5 = a circle fibration over CP^2
  - $ds^2_{S5} = ds^2_{CP2} + (dy + V)^2$, $dV = 2J$, $J = *J$, $y \sim y + 2\pi$
- S^5/Z_K ($y \sim y + 2\pi/K$) has no fixed point

- Hamiltonian = the conformal dimension operator H

- Superconformal index

$$Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}, S = Q^\dagger$$

Index Function on S^1 x S^5

- Supercharge $$Q^{R_1,R_2}_{j_1,j_2,j_3} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}, S = Q^\dagger$$

- BPS bound: $$E = j_1 + j_2 + j_3 + 2(R_1 + R_2)$$

- Index function:

$$I = \mathrm{Tr}\Big[(-1)^F e^{-\beta'\{Q,S\}} e^{-\beta\left(E - \frac{R_1+R_2}{2} - m(R_1 - R_2) + aj_1 + bj_2 + cj_3\right)}\Big], \; a+b+c=0$$

- Euclidean Path Integral of (2,0) Theory on S^1x S^5
  - S^5= S^1 fiber over CP^2 : -i ∂_y = KK modes

$$k \equiv j_1 + j_2 + j_3$$

  - Z_K modding keeps only k/K=integer modes

6d Abelian Theory (Fermion+ Scalar)

- on R x S^5, ……. (Could Include H=dB)

$$-\frac{i}{2}\bar{\lambda}\Gamma^M\hat{\nabla}_M\lambda - \frac{1}{2}\partial_M\phi_I\partial^M\phi_I - \frac{2}{r^2}\phi_I\phi_I$$

- gamma matrices Γ^M, ρ^a
- Symplectic Majorana $\lambda = -BC\,\lambda^*$, $\varepsilon = BC\,\varepsilon^*$
- Weyl: $\Gamma^7\lambda=\lambda$, $\Gamma^7\varepsilon = -\varepsilon$
- 32 supersymmetry

$$\begin{aligned}
\delta\phi_I &= -\bar{\lambda}\rho_I\epsilon = +\bar{\epsilon}\rho_I\lambda, \\
\delta\lambda &= +\frac{i}{6}H_{MNP}\Gamma^{MNP}\epsilon + i\partial_M\phi_I\Gamma^M\rho_I\epsilon - 2\phi_I\rho_I\tilde{\epsilon}, \\
\delta\bar{\lambda} &= -\frac{i}{6}H_{MNP}\bar{\epsilon}\Gamma^{MNP} + i\partial_M\phi_I\bar{\epsilon}\Gamma^M\rho_I - 2\bar{\tilde{\epsilon}}\rho_I\phi_I.
\end{aligned}$$

- additional condition on Killing spinor:

$$\hat{\nabla}_M\epsilon = \frac{i}{2r}\Gamma_M\tilde{\epsilon}, \quad \Gamma^M\hat{\nabla}_M\tilde{\epsilon} = 2i\epsilon, \qquad \tilde{\epsilon} = \pm\Gamma_0\epsilon.$$

Dimensional Reduction to CP^2

- define new variables so some of Killing spinors are y-independent
- New variables with twisting

$$\epsilon_{old} = e^{-\frac{y}{4}M_{IJ}\rho_{IJ}}\epsilon_{new},$$
$$\lambda_{old} = e^{-\frac{y}{4}M_{IJ}\rho_{IJ}}\lambda_{new},$$
$$(\phi_1 + i\phi_2)_{old} = e^{+(3+p)iy/2}(\phi_1 + \phi_2)_{new}$$
$$(\phi_4 + i\phi_5)_{old} = e^{+(3-p)iy/2}(\phi_4 + i\phi_5)_{new}.$$

$$M_{12} = -M_{21} = \frac{3+p}{2},\ M_{45} = -M_{54} = \frac{3-p}{2}$$
$$p = \cdots, -5, -3, -1, 1, 3, 5, \cdots.$$

$$\partial_y \to \partial_y + \frac{3i}{2}(R_1 + R_2) + \frac{ip}{2}(R_1 - R_2)$$

- y-independent supersymmetry: $Q = Q^{++}_{---}, S = Q^{--}_{+++}$
  - singlet ε_+, ε_- : 2 surviving supersymmetries

- kk mode = $-i\partial_y$

$$k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + \frac{p}{2}(R_1 - R_2),\ p = \text{odd integer}$$

5d Lagrangian

$$Q = Q^{++}_{---}, S = Q^{--}_{+++}$$

- Lagrangian on R x CP^2 with 2 supersymmetries for any p:

$$S = \frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\sqrt{|g|}\,\mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta - \frac{2i}{3}A_\rho A_\sigma A_\eta\Big)$$
$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I + \frac{1}{4}[\phi_I,\phi_J]^2 - 2\phi_I^2 - \frac{1}{2}(M_{IJ}\phi_J)^2 - i(3-p)[\phi_1,\phi_2]\phi_3 - i(3+p)[\phi_4,\phi_5]\phi_3$$
$$-\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda - \frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda] - \frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn} + \frac{1}{8}\bar\lambda M_{IJ}\rho_{IJ}\lambda\Big], \qquad (2.27)$$

- Supersymmetry Transformation

$$\delta A_\mu = +i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda, \quad \delta\phi_I = -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda,$$
$$\delta\lambda = +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 2\phi_I\rho_I\tilde\epsilon - M_{IJ}\phi_I\rho_J\epsilon.$$

- p/2=-1/2 : $k = j_1+j_2+j_3+ R_1+ 2R_2$
  - additional supersymmetries: Total 8 supersymmetries

$$Q^{+-}_{-++},\ Q^{+-}_{+-+},\ Q^{+-}_{++-} \quad \text{conjugates}$$

Coupling Constant Quantization

- Instanton number on CP2

$$\nu = \frac{1}{8\pi^2}\int_{\mathrm{CP}^2} \mathrm{Tr}(F\wedge F) = \frac{1}{16\pi^2}\int_{\mathrm{CP}^2} d^4x\sqrt{|g|}\ \mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

- Instantons represents the momentum K or energy K:

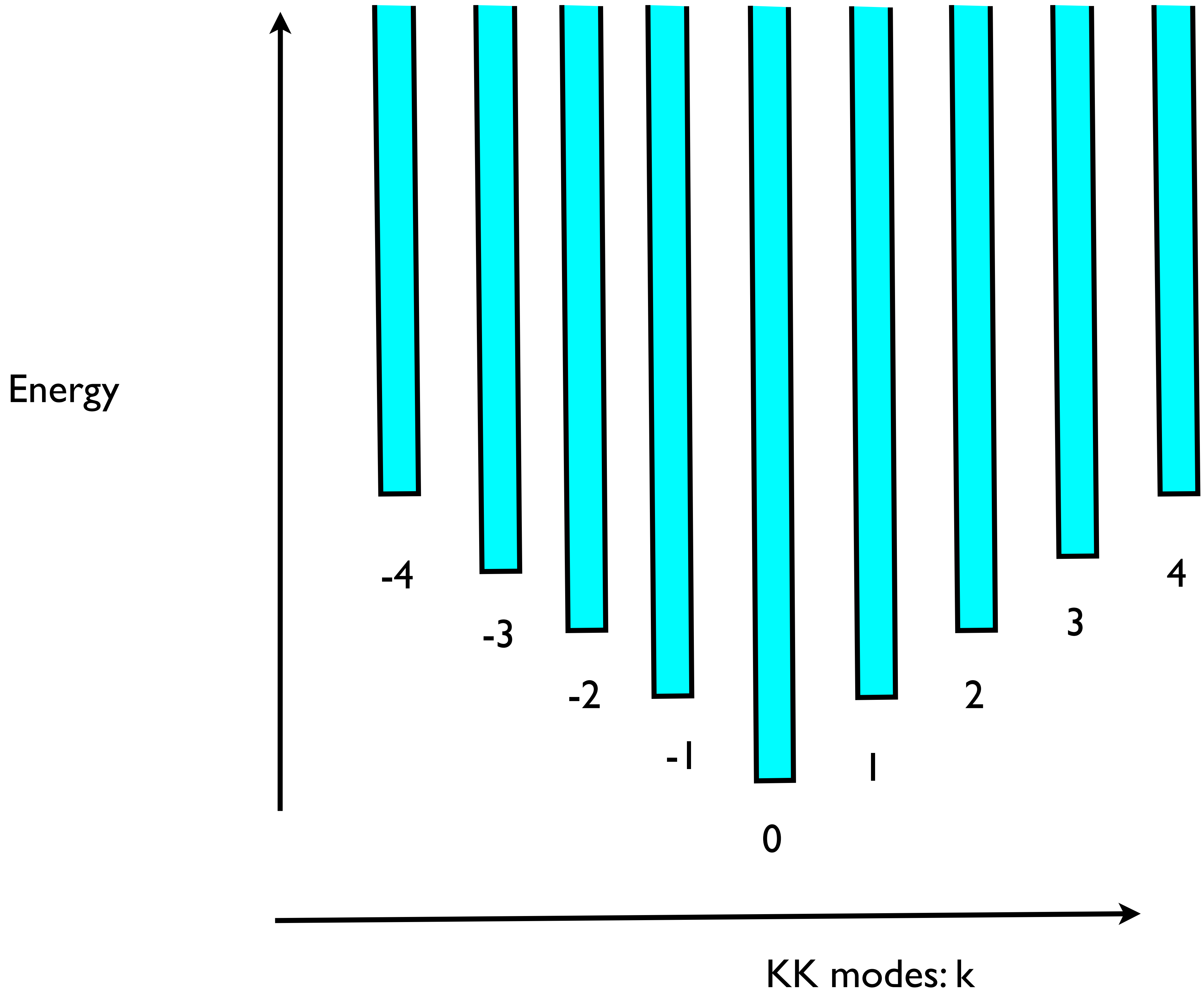
$$\frac{1}{g_{YM}^2} = \frac{K}{4\pi^2 r}$$

- Another approach to quantization: F=2J: 2π flux on a cycle, 1/2 instanton for abelian theory

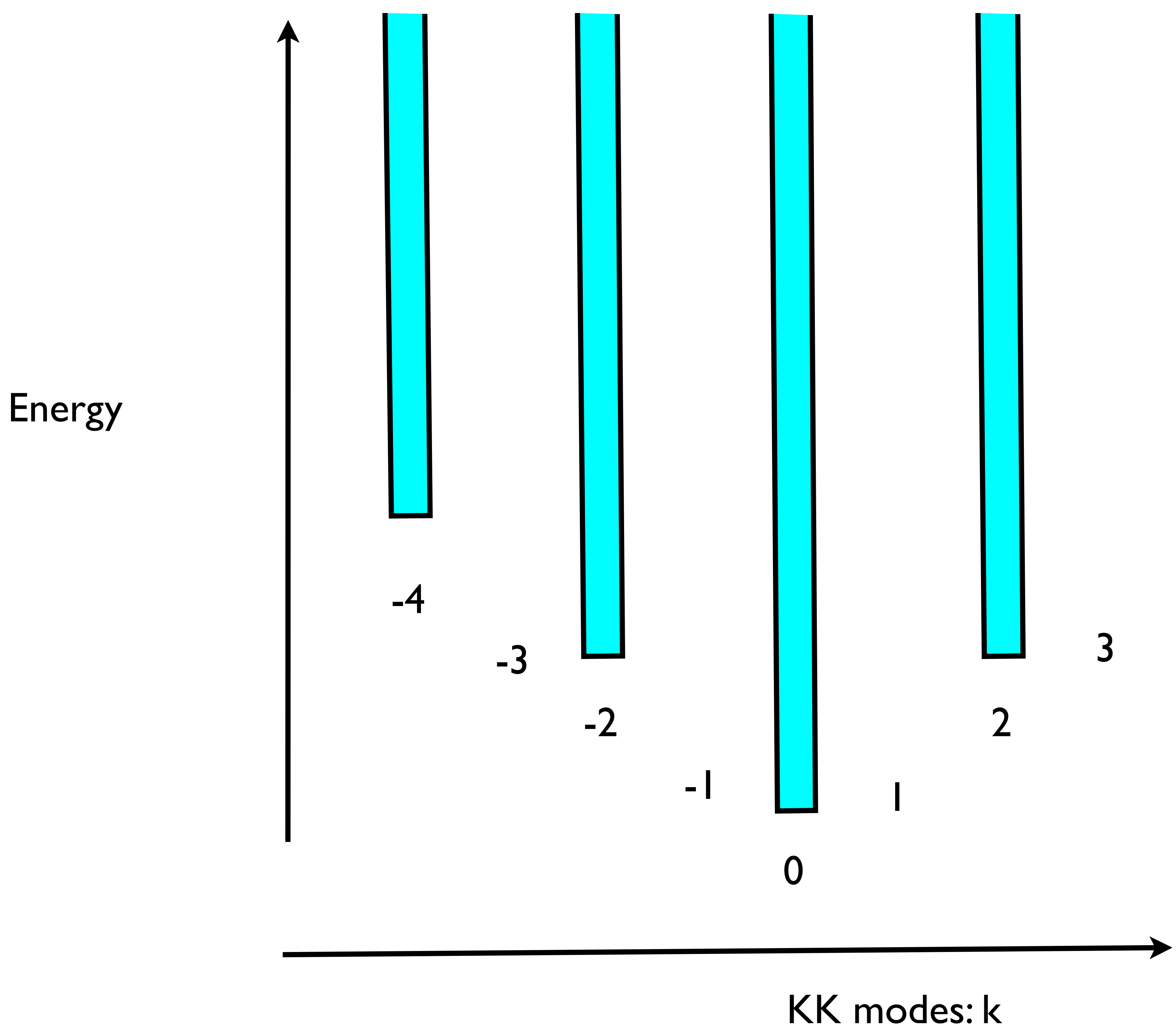
$$\frac{K}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\ \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\partial_\rho A_\sigma A_\eta \ \Rightarrow K\int dt\ A_0$$

- ‘t Hooft coupling constant: λ= N/K
- Large K => Free Theory

Diluting degrees of freedom with Z_K modding+ Twisting



Diluting degrees of freedom with Z_K modding+ Twisting



Expected Enhanced Supersymmetries

- Killing spinors with $p/2=-1/2$, $k = j_1+j_2+j_3+ R_1+ 2R_2$
  - $k=0$: 8 kinds
  - $k= \pm 1$: 14 kinds
  - $k= \pm 2$: 8 kinds
  - $k= \pm 3$: 2 kinds
- # of supersymmetries
  - $K \geqq 4$: 8 supersymmetries
  - $K=3$: 10 supersymmetries
  - $K=2$: 16 supersymmetries
  - $K=1$: 32 supersymmetries

the index function on S^1 x S^5

- 5d SYM on S^5 Hee-Cheol Kim, Seok Kim:1206.6339; Hee-Cheol Kim, Joonho Kim, S.K. 1211.0144
  - S-dual version of the index
  - Vacuum energy: $$(\epsilon_0)_{index} = \lim_{\beta'\to 0} \mathrm{Tr}\left[(-1)^F \frac{E-R}{2} e^{-\beta'(E-R_1)}\right] = \frac{N(N^2-1)}{6} + \frac{N}{24}$$
- S^1 x CP^2 path integral off-shell
- Stationary phase: $D^1=D^2=0$, $F= 2s J$, $\varphi + D^3=4s$, $s=\mathrm{diag}(s_1,s_2,\ldots,s_N)$
- Path Integral: Off-shell, localization

$$\sum_{s_1,s_2,\cdots s_N=-\infty}^{\infty} \frac{1}{|W_s|} \oint \left[\frac{d\lambda_i}{2\pi}\right] e^{\frac{\beta}{2}\sum_{i=1}^N s_i^2 - i\sum_i s_i\lambda_i} Z^{(1)}_{\rm pert} Z^{(1)}_{\rm inst} \cdot Z^{(2)}_{\rm pert} Z^{(2)}_{\rm inst} \cdot Z^{(3)}_{\rm pert} Z^{(3)}_{\rm inst} \,.$$

- For K=1, well-confirmed for k ≤ N with N=1,2,3 with the AdS/CFT calculation

Strange Vacua

* K=1, F=2sJ background

$$
\begin{aligned}
&U(2)\ (1,-1)\\
&U(3)\ (2,0,-2),(2,-1,-1),(1,1,-2),(1,0,-1)\\
&U(4)\ (3,1,-1,-3),(3,1,-2,-2),(2,2,-1,-3),(3,0,-1,-2),\\
&\qquad (2,1,0,-3),(2,0,0,-2),(2,0,-1,-1),(1,1,0,-2),(1,0,0,-1)
\end{aligned}
$$

* the Lowest one $s_G = 2\rho \cdot H$ with negative energy $-2\rho^2$, where ρ = Weyl vector
* Ground State for Index: K ≤ N (Strong 't Hooft coupling λ=N/K)

K	$U(2)$	$U(3)$	$U(4)$	$U(5)$	$U(6)$	$U(7)$	$U(8)$	$U(9)$	$U(N)$
1	-1	-4	-10	-20	-35	-56	-84	-120	$-\frac{N(N^2-1)}{6}$
2	0	-1	-2	-5	-8	-14	-20	-30	
3		0	-1	-2	-3	-6	-9	-12	
4			0	-1	-2	-3	-4	-7	
5				0	-1	-2	-3	-4	
6					0	-1	-2	-3	
7						0	-1	-2	
8							0	-1	
9								0	

Table 1: Vacuum energies divided by K, at general $\mathbb{Z}_K$ (and fluxes)

Check with AdS/CFT

- E.g. k = N = 3: (all results multiplied by vacuum energy factor & $e^{-3\beta}$) $y_i = e^{-\beta a_i}\ ,\quad y = e^{\beta(m-\frac{1}{2})}$

$$
\begin{aligned}
Z_{(2,0,-2)} &= 3\left[y^2(y_1+y_2+y_3)+y(y_1^2+y_2^2+y_3^2)+y^{-1}(y_1+y_2+y_3)-(1+\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots)+y^3\right] \\
&\quad +6y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right]+y^3 \\
Z_{(2,-1,-1)}+Z_{(1,1,-2)} &= -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -2y\left[y(y_1+y_2+y_3)-(y_1^{-1}+y_2^{-1}+y_3^{-1})+y^{-1}+y^2\right] \\
&\quad -4y^3-4y^2(y_1+y_2+y_3)-2y\left(y_1^2+y_2^2+y_3^2-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)+2\left(\frac{y_1}{y_2}+\frac{y_2}{y_3}+\cdots\right)-2y^{-1}(y_1+y_2+y_3) \\
Z_{(1,0,-1)} &= y^3+y^2(y_1+y_2+y_3)-y(y_1^{-1}+y_2^{-1}+y_3^{-1})+1
\end{aligned}
$$

add all

$$
Z_{SUGRA} = 3y^3+2y^2(y_1+y_2+y_3)+y\left(y_1^2+y_2^2+y_3^3-\frac{1}{y_1}-\frac{1}{y_2}-\frac{1}{y_3}\right)-\left(\frac{y_1}{y_2}+\frac{y_2}{y_1}+\cdots\right)+y^{-1}(y_1+y_2+y_3)
$$

- Non-zero flux states contributing to the index
  - s=(N-1,N-3,…,-(N-1)) = s_0 : SU(N) Weyl vector
  - index vacuum energy: $E_0 = -\dfrac{N(N^2-1)}{6}$

SU(2) Case

- BPS Eq. for Homogeneous Configuration with instanton number n^2

$$A = V\mathrm{diag}(n, -n), \;\; F = 2J\mathrm{diag}(n, -n),$$

- homogeneous solutions possible only with n=+1,-1
- but gauss law is violated
- for one of the constant bps solutions, the homogeneous fermionic zero mode is possible.
- gauss law can be satisfied with fermionic contribution for K=1 but not for K>1.
- energetic is more complicated to due to zero-point contribution to the classical one,...

6d (1,0) Theories

- Similar approach on R x CP2 works for some theories
- It would be interesting to check some index functions

- For other (1,0) 6d theories with instanton strings, it is not clear whether the corresponding 5d theories capture KK modes via instantons
  - Again we need to understand the instanton physics better

Conclusion

- New 5d supersymmetric theories for M5 are found with discrete coupling constant
- Index Function of 6d A_N (2,0) is partially obtained.
- highly nontrivial vacuum structure in the strong coupled regime

- UV finite? How rigid is the theory with eight supersymmetries.
- Enhanced supersymmetry to K=1,2,3?
- Wilson-loop can be included.
- Near BPS objects? perturbative approach?
- Formulation and Calculation of 6d N=1 SCFT Theories
- Relate our result to Rastelli's result?

5d $\mathcal{N}$=2 SYM as the M5 brane theory

- compactification on $R^{1+4} \times S^1$ with radius r
- the lowest KK modes ⇒ 5d SYM
- coupling constant $1/g_{YM}^2 = 4\pi^2/r$
- instanton = quantum of KK modes of unit momentum
- drop KK modes and keep instantons
  - otherwise, it is over-counting
  - there may quantum-gauge-invariance identifying two
  - dyonic instanton index
  - monopole string+ momentum

Hee-Cheol Kim, Seok Ki, E. Koh, KL, Sungjay Lee 2011

Haghighat, Iqbal, Kozcaz, Lockhart, Vafa: M-strings

- 5d SYM + instantons = ? 6d (2,0) theory
  - 6-loop UV divergence in four-point function

Z. Bern, J. J. Carrasco, L. J. Dixon, M. R. Douglas, H. Johansson, M. von Hippel